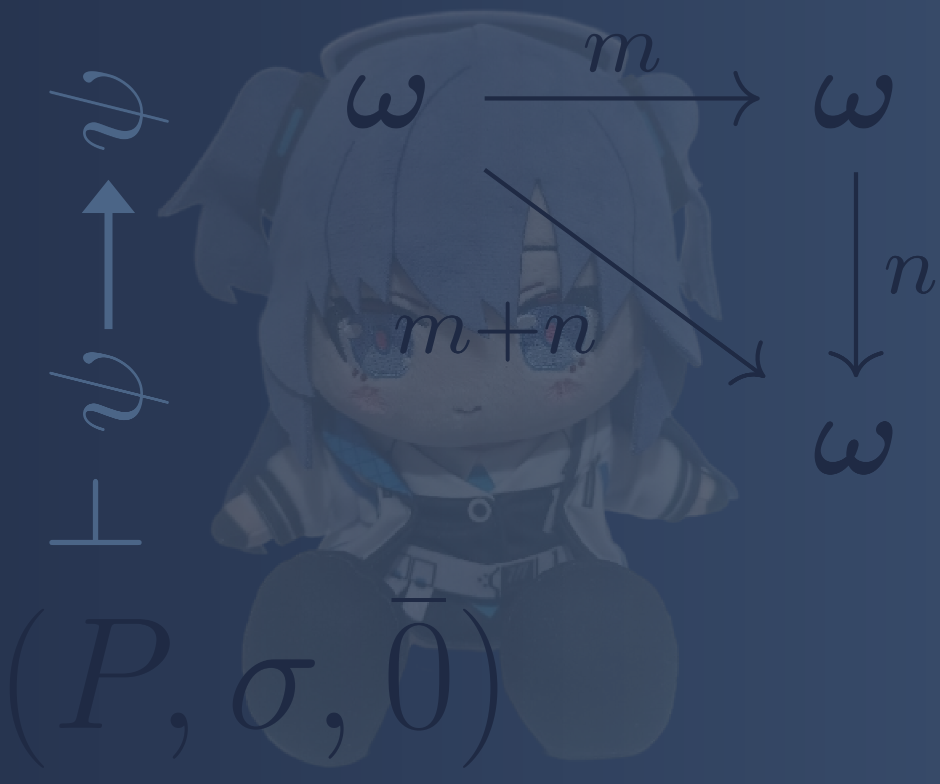
Hayase Yuuka's notes on:

FOUNDATIONS OF MATHEMATICS



Logic & Reason Series

Hayase

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idk

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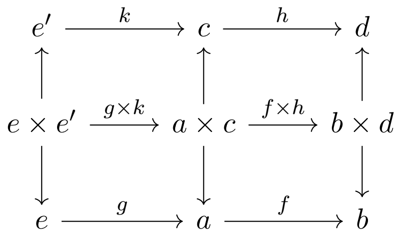
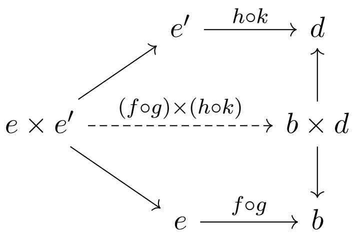
Exercise 8. (ii). Prove that $(f \times h) \circ (g \times k) = (f \circ g) \times (h \circ k)$



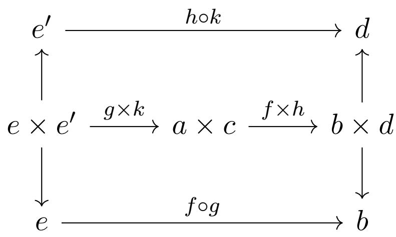
Proof. Consider that there's a unique arrow $(f \circ g) \times (h \circ k) : e \times e' \longrightarrow b \times d$ such that the diagram:



commutes. Now, for the first diagram,

$$\begin{aligned} \pi_d \circ (f \times h) \circ (g \times k) &= \pi_d \circ \langle f \circ \pi_a, h \circ \pi_c \rangle \circ \langle g \circ \pi_e, k \circ \pi_{e'} \rangle \\ &= h \circ \pi_c \circ \langle g \circ \pi_e, k \circ \pi_{e'} \rangle \\ &= h \circ k \circ \pi_{e'} \end{aligned}$$

And in a similar way $\pi_b \circ (f \times h) \circ (g \times k) = f \circ g \circ \pi_e$, so the diagram:



commutes. By the uniqueness of $(f \circ g) \times (h \circ k)$ we have that $(f \times h) \circ (g \times k) = (f \circ g) \times (h \circ k)$.

□